

Modelling the solar, geomagnetic and periodic variations of the ionosphere over Ethiopia

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A careful and continuous ionospheric modelling can significantly influence the performance of activities such as Positioning, Navigation and Timing services related with the Global Navigation Satellite System applications as well as the Earth Observations System, satellite communication and Space weather forecasting applications. In this paper, the linear time-series modelling that consists of the solar, geomagnetic and periodic components has been carried out on the daily ionospheric $vTEC$ at two different Ethiopian GPS locations, at Arbaminch, ARMI (geographic 6.06° N, 37.56° E) and Bahir Dar, BDMT (geographic 11.60° N, 37.38° E), for the year 2012, 2014 and 2016 in the 24th solar cycle. The variations of $vTEC$ due to the solar activities, geomagnetic activities and periodic oscillations have been explicitly investigated. The results confirmed that the correlation coefficient of the linear model based estimated $vTEC$ and the observed GPS- $vTEC$ is around 80% in the year 2014. Besides, solar activity is identified as the key component for the 27 days period variations of $vTEC$ whereas geomagnetic activity is identified as the key component that influences the short-period variations of the daily average $vTEC$. In addition to the correlation analysis, the accuracy of the model has been assessed by comparing the International Reference Ionosphere (IRI 2016) model based $vTEC$ and GPS- $vTEC$ measurements as well as with the quadratic model based $vTEC$. Consequently, the linear model formulated with the solar, geomagnetic and periodic components significantly captured the variations (78–80%) of the observed $vTEC$ compared with both the IRI 2016 and the quadratic models during the years 2012, 2014 and 2016. The comparison of the observed and predicted $vTEC$ variations has also been examined using the continuous wavelet transform. The decomposed waves from the wavelet analysis have revealed that the predicted and observed $vTEC$ have had simultaneous periods of variations specifically with the period of 27 days whereas the IRI 2016 could capture the short-period variations of $vTEC$. Moreover, the analysis from the transformed data in the year 2014 over both Arbaminch and Bahir Dar has indicated that the linear model based $vTEC$ and the observed GPS- $vTEC$ have had common pattern of variations with the period of 27 days that had lasted for 150 days (from day of the year 100 to 250).

Key words: ionospheric variability, equatorial ionosphere, solar induced ionospheric variability

INTRODUCTION

Ionosphere, which can be defined as the ionised part of the upper Earth atmosphere, is significantly varying due to the solar and geomagnetic activities. This variability can be traced from the observations and empirical/physical models. We commonly use the ionospheric total electron content (TEC) and electron density to extract this variability. Different ionospheric models have been proposed for the ionospheric TEC variability and are formulated by considering the solar, geomagnetic and periodic components [9, 11, 14, 15, 17, 18, 25, 26, 31, 40]. In [17] the empirical ionospheric model was formulated by considering the solar $F10.7$ index. The solar activity and seasonal components are identified by these investigators as the prime contributors to the F-region

ionospheric variability.

The linear regression-based global ionospheric TEC model was proposed by Afraimovich in [2] with the intention of demonstrating the relation between global TEC and solar activity component, $F10.7$ index. They reported that the Global Electron Content (GEC) and $F10.7$ series regression are well approximated by a linear dependence. However, the limitation was that the global TEC models do not consider the local characteristics and seasonal variations [19, 20, 22, 24].

Productive modifications have been done by [37] to this linear global TEC model by including the periodic component that describes the seasonal dependence of TEC. As a result, the modified model has demonstrated better performance than the model

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proposed by [1, 3], even in the absence of the effect of geomagnetic component.

In [6] and [40] it was reported that the variability of the ionosphere increases with geomagnetic activity at all latitudes. Yu et al. [39] have studied the climatological variations of ionospheric TEC with a new perturbation index (σ) as a function of the solar $F10.7$ and geomagnetic Ap indices. Accordingly, the global perturbation index correlated well with $F10.7$ and Ap indices. Similarly, in [29], [30] and [35] the different factors affecting the TEC climatological variations at different latitudes have been investigated using the Global Ionospheric Map (GIM)-TEC data based linear time series TEC model. These models were not evaluated with the standard ionospheric empirical models. The GIM model TEC may not be appropriate for characterising the low-latitude ionospheric variations [30].

Recently, significant reports were compiled on the GPS-based ionospheric TEC variations studies and their comparison with the empirical models such as IRI and NeQuick estimations over equatorial and low latitude regions across India [7, 28, 36]. Consequently, concert climatological improvements in the global empirical models have been achieved with these research findings. More specifically, attempts have also been made by numerous researchers to conduct the comparative studies between the GPS-TEC and IRI-based TEC over the equatorial and low latitude African and American longitudes [5, 34, 38]. They have figured out the drawbacks of IRI model predictions specifically during the early morning and post-sunset hours for both solar minimum and maximum conditions.

The intention of the current work is to investigate the low latitude ionospheric vTEC variations using the linear time series vTEC model by incorporating three forms of variations, namely the solar activity ($F10.7P$), geomagnetic activity (Ap) and periodic variations, simultaneously. We also attempted to analyze the periodic variations at two different frequencies, namely the Semiannual Oscillation (SAO) and the Annual Oscillations. In the meantime, the dominant component of the variation of vTEC is identified. The validation of the linear time series model is done by comparing the model vTEC with the GPS-vTEC. Besides, to demonstrate the accuracy of the proposed linear time series model, we have compared the IRI 2016 and quadratic models-based vTEC with the GPS-TEC values. The model parameters are estimated over Arbamich, ARMI (geographic 6.06° N, 37.56° E), Ethiopia in the year 2014 and the validation of the linear model is conducted over another Ethiopian low latitude GPS station, Bahir Dar, BDMT (geographic 11.60° N, 37.38° E), for the year 2014. Consequently, forecasting/prediction of the

daily vTEC has been done over Bahir Dar for the years 2012 and 2016.

DATA DESCRIPTION

In this work, the performance of the linear time series vTEC model was validated in characterising the ionospheric GPS-vTEC for the year 2014 at two low-latitude GPS stations in Ethiopia, located at Arbaminch (geographic 6.06° N, 37.56° E) and Bahir Dar (geographic 11.6° N, 37.38° E). Ionospheric vTEC were collected using the ground-based GPS receivers deployed at these two stations and the data were obtained from Scripps Orbit and Permanent Array Center Archive¹. After estimating the model parameters, we used them to predict the vTEC and validation was performed with the GPS-vTEC values over Bahir Dar in the year 2012 and 2016. The detailed coordinates of the GPS receivers used in this study are presented in Table 1.

The daily mean vTEC values of each day were obtained from the corresponding hourly mean vTEC observations, which are in turn inferred from the mean of vTEC values of all visible satellites in each 5 min interval. In this study, the approach communicated by [29] was implemented for the realisation of the proposed linear time series model.

In this linear model, we have used the solar EUV irradiance ($F10.7p$), geomagnetic activity (Ap) and periodic oscillations (annual and semiannual oscillations) as the model input components. The daily variations of model inputs, $F10.7$ and Ap index, in the year 2014 are shown in the top and bottom panels of Fig. 1 respectively. Since the units of solar and geomagnetic activity indices differ from the unit of vTEC, the coefficients of each parameter in the proposed linear model are normalised as unitless values [2, 9]. The daily mean vTEC values, solar activity index ($F10.7$) and geomagnetic index (Ap) were normalised using Equations (1), (2) and (3), respectively:

$$NvTEC(d) = \frac{vTEC(d)}{\max(vTEC(d))}, \quad (1)$$

$$F10.7_a = \frac{F10.7(d)}{\max(F10.7(d))}, \quad (2)$$

$$Ap_a = \frac{Ap(d)}{\max(Ap(d))}, \quad (3)$$

where $NvTEC(d)$ is the normalised vTEC value, $F10.7_a$ is the normalised solar proxy, Ap_a is the normalised geomagnetic proxy and d indicates the day respectively. The idea of Eq. (1) is used in the conversion of the estimated linear model based normalised vTEC to the actual TEC unit.

¹<http://sopac.ucsd.edu>

Table 1: Coordinates of the GPS receivers used for the study

Station	Code	Geographic	Geomagnetic
		Lat.(N), Long.(E)	Lat.(N), Long.(E)
Arbaminch	ARMI	(6.06,37.56)	(2.62,110.87)
Bahir Dar	BDMT	(11.60,37.38)	(8.08,111.57)

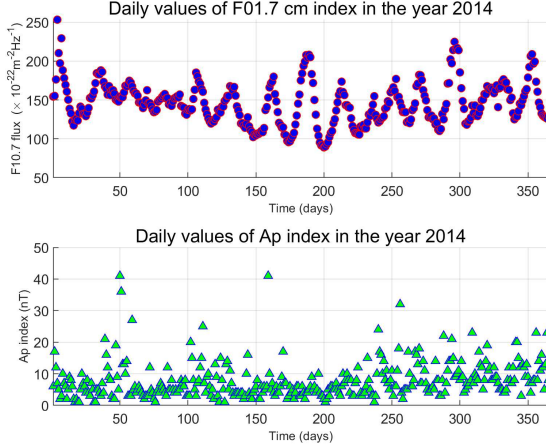


Fig. 1: Inputs used in the linear time series model in the year 2014. The top panel illustrates the daily variations of solar $F10.7$ cm flux and the bottom panel demonstrates the average planetary index (A_p index).

TEC FROM THE DUAL FREQUENCY GPS RECEIVER

The receivers at GPS stations record signals transmitted at two L-band frequencies, namely f_1 at 1575.42 MHz and f_2 at 1227.60 MHz. The time delay which occurs while these signals are propagating through the ionosphere is converted to the pseudo-ranges and recorded as the P_1 and P_2 signals. The carrier phase delay measurements on the f_1 and f_2 coherent frequencies are also recorded as L_1 and L_2 , respectively. The delayed and phase shifted signals are recorded in a special format called the Receiver Independent Exchange Format (RINEX). The time delays of signals are converted to the pseudo-range values and the phase shifts are recorded as the phase delays in the receivers [16, 18, 27, 33]. The standard model for pseudo-range recordings for two frequencies f_1 and f_2 is as follows:

$$P_{1,u}^m = \rho_u^m + c(\delta t_u - \delta t^m) + d_{trop,u}^m + d_{ion1,u}^m + c(\epsilon_1^m + \epsilon_{1,u}) \quad (4)$$

$$P_{2,u}^m = \rho_u^m + c(\delta t_u - \delta t^m) + d_{trop,u}^m + d_{ion2,u}^m + c(\epsilon_2^m + \epsilon_{2,u}), \quad (5)$$

where the subscript u denotes the receiver station index; the subscript m denotes the satellite index. ρ is the actual range between satellite and receiver, δt_u and δt^m are the clock errors for the receiver and satellite, respectively. d_{trop} and d_{ion} are the troposphere and ionosphere group delays, respectively. ϵ^m and ϵ_u are the frequency-dependent satellite and receiver biases. The model for GPS recordings also includes the antenna pattern and noise errors. Since those are assumed to be the same for both frequencies, usually they are not used in the model equation for TEC. c is the speed of light in vacuum.

The difference of Equations (4) and (5) is called the geometry free linear combination of pseudo-range because the actual range ρ is eliminated as

$$P_{4,u}^m = P_{2,u}^m - P_{1,u}^m = d_{ion2,u}^m - d_{ion1,u}^m + c(\epsilon_2^m - \epsilon_1^m) + c(\epsilon_{2,u} - \epsilon_{1,u}). \quad (6)$$

The tropospheric contribution $d_{trop,u}^m$ in Equations (4) and (5) and any other source of error are also eliminated since they are not a function of frequency. Using the satellite and receiver biases for f_1 and f_2 frequency signals, the interfrequency or differential code biases (DCBs) are defined for the satellite and receiver as follows:

$$DCB^m = \epsilon_1^m - \epsilon_2^m \quad (7)$$

$$DCB_u = \epsilon_{1,u} - \epsilon_{2,u}, \quad (8)$$

where DCB^m and DCB_u are the differential code biases for the satellite and receiver, respectively.

Similar equations can be written for the phase delay observations $L_{1,u}^m$ and $L_{2,u}^m$ as:

$$L_{1,u}^m = \lambda_1 \Phi_{1,u}^m = \rho_u^m + c(\delta t_u - \delta t^m) + \lambda_1 \Phi_{ion1,u}^m + \lambda_1 \Phi_{trop,u}^m - c(\epsilon_1^m + \epsilon_{1,u}) + \lambda_1 N_1^m \quad (9)$$

$$L_{2,u}^m = \lambda_2 \Phi_{2,u}^m = \rho_u^m + c(\delta t_u - \delta t^m) + \lambda_2 \Phi_{ion2,u}^m + \lambda_2 \Phi_{trop,u}^m - c(\epsilon_2^m + \epsilon_{2,u}) + \lambda_2 N_2^m, \quad (10)$$

where λ_1 and λ_2 are the wavelengths corresponding to f_1 and f_2 frequencies, $\Phi_{ion1,u}^m$ and $\Phi_{ion2,u}^m$ are the ionospheric phase delays corresponding to f_1 and f_2 frequencies, respectively. N_1^m and N_2^m denote the initial phase ambiguity corresponding to the f_1 and f_2 frequencies, respectively for the m^{th} satellite. Finally, $\Phi_{trop,u}^m$ is the phase delay due to the troposphere.

The difference of Equations (9) and (10) is called the geometry free linear combination of phase delays and is given by

$$L_{4,u}^m = \lambda_1 \Phi_{1,u}^m - \lambda_2 \Phi_{2,u}^m = \lambda_1 \Phi_{ion1,u}^m - \lambda_2 \Phi_{ion2,u}^m + c(\text{DCB}^m) + c(\text{DCB}_u) + \Delta N^m, \quad (11)$$

where ΔN^m in Eq. (11) is defined as

$$\Delta N^m = \lambda_1 N_1^m - \lambda_2 N_2^m. \quad (12)$$

Using the approximation given by [16, 18, 27, 33]:

$$d_{ion,u}^m = -\Phi_{ion1,u}^m \frac{c}{f} \approx A \frac{s\text{TEC}_u^m}{f^2}, \quad (13)$$

where $A = 40.3 \text{ m}^3/\text{s}^2$ and $s\text{TEC}_u^m$ denotes the total electron content on the slant ray path combining the receiver u and the satellite m . Using Equation (13) in Equations (6) and (11), the following expressions for the geometry free combinations are obtained:

$$P_{4,u}^m = A \left(\frac{f_1^2 - f_2^2}{f_1^2 f_2^2} \right) s\text{TEC}_u^m -$$

$$L_{4,u}^m = A \left(\frac{f_1^2 - f_2^2}{f_1^2 f_2^2} \right) s\text{TEC}_u^m - c(\text{DCB}^m + \text{DCB}_u) \quad (14)$$

$$- c(\text{DCB}^m + \text{DCB}_u) + \Delta N^m. \quad (15)$$

For a selected measurement time, slant Ray Total Electron Content ($s\text{TEC}$) can be calculated using either pseudo-range or carrier phase data from each satellite. $s\text{TEC}$ calculated from Eq. (14) is noisy and open to multipath effects:

$$s\text{TEC}_u^m(n) = \frac{1}{A} \left(\frac{f_1^2 f_2^2}{f_1^2 - f_2^2} \right) [P_{4,u}^m(n) + c(\text{DCB}^m + \text{DCB}_u)], \quad (16)$$

where the index n denotes the time sample and $1 \leq n \leq N$.

In order to obtain the $s\text{TEC}$ from Equation (15)), the initial ambiguity ΔN^m needs to be resolved. In the works [16, 18, 27, 33] the following baseline method is used: first, a baseline B for each connected arc is obtained by differentiating the pseudo-range and phase measurements as:

$$B = \frac{1}{N_{me}} \sum_{n_{me}=1}^{N_{me}} (P_{4,u}^m(n_{me}) - L_{4,u}^m(n_{me})), \quad (17)$$

where N_{me} is the number of measurements in a connected phase arc. Then the slant TEC can be computed by inserting B into the phase Equation (15)) and the $s\text{TEC}$ can be extracted as

$$s\text{TEC}_u^m(n) = \frac{1}{A} \left(\frac{f_1^2 f_2^2}{f_1^2 - f_2^2} \right) (B + L_{4,u}^m(n) + c(\text{DCB}_u + \text{DCB}^m)). \quad (18)$$

In the above Equations u and m denote the receiver and satellite id's, respectively, n is the measurement time. L_4 is the geometry free linear combination of carrier phase data and B is the baseline value defined in Equation (17). DCB_u and DCB^m are the receiver and satellite differential code biases respectively. In Eq. (18) P_4 is the pseudorange geometry free linear combination for dual frequency GPS signals. Each cycle slip or phase disconnection starts another baseline calculation. Once the slant TEC is computed, the vertical TEC, $v\text{TEC}$, can be obtained using the thin shell approximation of the Single Layer Ionosphere Model (SLIM) as

$$v\text{TEC}_u^m(n) = \frac{s\text{TEC}_u^m(n)}{M(\epsilon_m(n))}, \quad (19)$$

where $M(\epsilon_m(n)) = [1 - (R \cos \epsilon_m(n)/(R + h))^2]^{-1/2}$ is called the mapping function and ϵ is the satellite elevation angle. R is the Earth radius of 6378.137 km and h is the ionospheric shell height of 428.8 km.

METHODS OF ANALYSIS

Ionospheric linear TEC model can be addressed component-wise with respect to each model parameter. In accordance with their relative contributions, the linear combination of these components will give us:

$$Nv\text{TEC}(d) = \text{TEC}_a(d) + \text{TEC}_{sol}(d) + \text{TEC}_{osc}(d) + \text{TEC}_{geo}(d), \quad (20)$$

where $\text{TEC}_a(d)$ is a constant representing additional variations arising from the solar activity. Consequently, $\text{TEC}_{sol}(d)$ is the solar activity component, $\text{TEC}_{osc}(d)$ is the periodic component at different frequencies and $\text{TEC}_{geo}(d)$ is the geomagnetic component.

Since the solar activity $F10.7$ radio flux differs greatly in the aspects of day-to-day and monthly variations, $F10.7_P$ index was considered in the proposed linear model to represent the solar activity influence on TEC [31]. The $F10.7_P = (F10.7_a + F10.7_{p81})/2$ could be determined on basis of the daily averaged solar activity index $F10.7_a$ and its 81-day running average $F10.7_{p81}$. According to [28, 30, 31, 36], the effect of solar EUV flux on ionospheric TEC can be sufficiently addressed by using the $F10.7_P$ index in the model.

The solar component of the linear time series model can be expressed as

$$\text{TEC}_{sol}(d) = \beta_2 F10.7_P(d-1) + \beta_3 F10.7_{p81}(d-1), \quad (21)$$

where $(d-1)$ is the lag of one day and β_2 and β_3 are the unknown coefficients indicating the magnitude of the solar component variations and are supposed to be estimated using the Bayesian-based parameter estimation technique. The effect of geomagnetic component on the daily mean TEC values can be represented as part of the linear model as follows:

$$\text{TEC}_{geo} = \beta_4 Ap(d-1) + \beta_5 Ap(d-2) + \beta_6 Ap(d-3),$$

where $(d-2)$ and $(d-3)$ are the lag 2 days and 3 days in Ap indices, respectively, and $\beta_4, \beta_5, \beta_6$ are the unknown coefficients representing the weight of the geomagnetic effects. A combination of solar modulated sinusoidal terms can be used to express the influence of periodic oscillations at different periods (182.6 and 365.25) as shown below:

$$\text{TEC}_{osc}(d) = \beta_7 \sin\left(\frac{2\pi d}{182.6}\right) + \beta_8 \cos\left(\frac{2\pi d}{182.6}\right) + \beta_9 \sin\left(\frac{2\pi d}{365.25}\right) + \beta_{10} \cos\left(\frac{2\pi d}{365.25}\right),$$

where the periodic component is a function of sine and cosine terms to resolve the phase and amplitude of each oscillation. The periods of 182.6 and 365.25 days represent the semiannual oscillation (SAO) and the annual oscillation (AO) respectively, β_7 to β_{10} are the unknown coefficients to be estimated.

The unknown coefficients $\beta_1, \beta_2, \dots, \beta_{10}$ were computed using the Bayesian-based parameter estimation technique. Model (predicted) vTEC values were estimated by using the model coefficients (parameters) obtained from inversion of the GPS-vTEC data from Arbaminch and Bahir Dar stations during the year 2014. Mean of the distribution of the estimated model parameters and the solar, geomagnetic and periodic inputs are used to accomplish the model prediction (See Fig. 2 for the distribution of the estimated model parameters).

The deviations between the observed GPS-TEC values and the model TEC values are also analysed by computing the root mean square error (RMSE) which is given by:

$$\text{RMSE} = \sqrt{\sum_{i=1}^n (X_i - Y_i)^2 / n},$$

where X_i represents the observed GPS-TEC data, Y_i can be the model based predicted TEC data of either the proposed linear model or the IRI 2016 model or the quadratic model.

In this work we have used the continuous wavelet transformation to explicitly demonstrate the common periodic variations of the observed GPS-vTEC

and the predicted (linear model based) vTEC daily values. Continuous Wavelet Transform (CWT) of a time series signal (data) given by $f(t)$ is defined as the sum over all time of the signal multiplied by the scaled and shifted version of the wavelet function $\psi_{s,\tau}(t)$. The equation:

$$\gamma_{s,\tau}(t) = \int f(t) \psi_{s,\tau}^* dt,$$

where $*$ denotes the complex conjugation, shows how a function $f(t)$ is decomposed into a set of basis functions $\psi_{s,\tau}(t)$ called the wavelets. The variables s and τ are the new dimensions, scale and translation (position), after the wavelet transform. Scaling a wavelet simply means stretching (or compressing) it, whereas translating a wavelet means delaying (or hastening) its onset.

The wavelets are generated from a single basic wavelet $\psi_{s,\tau}(t)$, the so-called mother wavelet, by scaling and translation:

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right),$$

where the factor $s^{-1/2}$ is for energy normalisation across the different scales [12, 13, 21, 23, 32].

The International Reference Ionosphere (IRI) is an international empirical standard model used for the specification of ionospheric parameters. The model provides average values of the electron density, electron content, electron and ion temperature and ion composition as functions of the height, location, local time and sunspot number for magnetically quiet conditions. To enhance the capacity of model the improvements have been made through the ingestion of all worldwide available data from the ground-based as well as satellite observations. As a result, a new version of the model (IRI 2016) has been released in 2017 by incorporating some new input parameters that are supposed to increase its capacity. The IRI 2016 model includes two new model options for the F2-peak height $h_m F_2$ and a better representation of the topside ion densities at the very low and high solar activities.

Two new options are used in modelling of $h_m F_2$ directly and no longer through its relationship to the propagation factor $M(3000)F_2$. Thus, the new model options enable the IRI 2016 model² to predict the evening peaks, this was not possible in the old versions. In addition, the improvement of the ion composition model in the topside ionosphere can lower the transition height from close to 1000 km down to almost 600 km in the new version of the model. A number of smaller changes have also been made concerning the use of solar indices and the speed-up of the computer program [11]. For a given location, time and date, like the previous versions of

²<http://omniweb.gsfc.nasa.gov/vitmo/iri-vitmo.html>

the model, the IRI 2016 model provides the monthly averages of ionospheric parameters (such as TEC) in the altitude range of about 50–2000 km.

According to [31] and [29], the quadratic model of the mean daily ionospheric vTEC as the function of either solar ($F10.7$ flux) or geomagnetic (A_p index) components can be formulated as

$$\text{vTEC}(F10.7) = c + b \times F10.7(d) + a \times F10.7(d)^2.$$

RESULTS AND DISCUSSION

We have used the comparison of the IRI 2016 model based vTEC and the quadratic model based daily vTEC values with their corresponding observed GPS-vTEC values to validate the performance of a linear time series model in terms of its capacity of capturing the ionospheric vTEC variations during the year 2014 at two geographic locations (Arbaminch and Bahir Dar) in Ethiopia. Once the model is validated, we implemented the model parameters to predict the vTEC values during the years 2012 and 2016 over Bahir Dar, Ethiopia ionosphere. The daily averaged vTEC at Arbaminch and Bahir Dar stations were considered for the analysis. The main findings of this work are presented in the following subsections.

The distribution and trace plots for the estimated parameters over Arbaminch are displayed in Figs. 2 and 3 respectively. In these figures β_2 and β_3 refer to the variation due to the solar component; β_4 to β_6 due to the geomagnetic component and β_7 to β_{10} due to the periodic components. The influence of individual components on the daily averaged TEC values at Arbaminch is inferred from the width (thickness) of either Fig. 2 or Fig. 3. Accordingly, the geomagnetic component was the key component that influenced the daily averaged TEC.

The periodical variations of the $F10.7$ flux, A_p index and vTEC are portrayed by the left, middle and right panels of Figures 10 and 11, respectively. The variations of vTEC due to the 27 days solar rotation period, as can be inferred from left and right panels of the figures 10 and 11, are dominantly affected by the solar component.

In contrast to this, the short-period variations of TEC, as illustrated by the middle panels of these figures, are significantly influenced by the geomagnetic component. These observations are against the reports [17] and [29] that have attempted to conclude that the solar component is totally the dominant component influencing the daily average vTEC values. Such deviated observation might be due to a difference in instrumentation as well as the latitudinal and wind dynamics differences [8–10].

The top and bottom panels of Figures 4 and 5 illustrate the linear correlation between the observed, IRI 2016 model based and linear model based vTEC ($\text{TECU} = 10^{16} \text{ele}/m^2$) values over Arbaminch and Bahir Dar respectively in the year 2014. The linear

model based vTEC is nicely correlated with the observed vTEC with the average correlation coefficient of 0.8 with the RMSE of 14 and 33 over Arbaminch and Bahir Dar respectively. This implies that 80% of the variation of vTEC is captured by the proposed linear model during the year 2014.

We have also noticed that the double peak structures in the GPS-vTEC and the linear model-vTEC values were similar to the semiannual variation (see the top panels of Figures 4 and 5). The semiannual variation of TEC could also be due to the combined effect of the solar zenith angle and magnetic field geometry [28, 30, 31, 36].

Figures 6 and 7 depict the common periodical variations of the observed and linear model based vTEC over Arbaminch and Bahir Dar respectively. The left and right panels of these figures demonstrate the wavelet transformed variations showing the periodical variations of the observed and modeled vTEC values over Arbaminch and Bahir Dar in the year 2014. These transformed variations have revealed that the predicted and observed vTEC have varied simultaneously with the periods of 27 days. These common variations were pronounced during the days of the year from 100 to 250 (referring to the x-axes of Figures 6 and 7). Comparable observations were made by [32] and [23] at different latitudinal and longitudinal sectors.

Figures 8 and 9 are devoted to the illustration of whether the IRI 2016 model has captured the possible variations of vTEC or not. The left panel stands for the observed vTEC and the right panel for the IRI 2016 over both Arbaminch and Bahir Dar. As one can see from these figures, the variations of GPS-vTEC due to the solar period of 27 days (left panel), which were nicely captured by the proposed model, were not addressed by the IRI 2016 model. This signifies the quality of the current linear model formulated with the solar, geomagnetic and periodic components against the IRI 2016 model in capturing the expected daily vTEC variations. However, the variations of GPS-vTEC with short periods (such as the period of 5 days) have been explained better by the IRI 2016 model compared to the current linear model.

The estimated parameters for the proposed linear model, using the daily GPS-vTEC data over Arbaminch in the year 2014, are implemented to forecast the daily values of vTEC in the years 2012 and 2016 over Bahir Dar ionosphere. The correlations between the forecasted and observed vTEC values during the years 2012 and 2016 over Bahir Dar are shown by the scatter plots in Figures 16 and 17 respectively. The corresponding deviations of the linear model based vTEC from the observed vTEC for the years 2012 and 2016 are demonstrated by the top and bottom panels of Figures 19 and 20. Similarly the deviation of the quadratic model based vTEC values from the observed GPS-vTEC daily values during the year

2016 is depicted by the top and bottom panels of Fig. 21. The linear correlation coefficients between the observed and linear model based vTEC values are 0.7 for the year 2012 and 0.78 for 2016 whereas the correlation of the observed vTEC with quadratic model based vTEC is about 0.4 for the year 2016 (Fig. 18). This means that 78% of the variations of GPS-vTEC have been described by the proposed linear model whereas the quadratic model has captured only 40% of it. This also signifies the strength of the linear time series model in describing the expected periodical, solar and geomagnetic activity related variations to the optimal stage.

We have analysed the seasonal dependence of performance of the proposed linear model by considering the December Solstice, the June Solstice, the March Equinox and the September Equinox observations over Bahir Dar in the year 2014. The correlations between the observed and linear model based vTEC during these seasons are illustrated by Figures 12, 13, 14 and 15 with the correlation coefficient values of 0.6, 0.9, 0.5 and 0.8, respectively. This implies that there is a clear seasonal variation of performance of the proposed linear model. The seasonal variations of performance of the model might be associated with the effect of wind dynamics [4, 5, 25, 26].

CONCLUSIONS

The aim of this study was to validate the performance of the linear time series model, formulated with the solar, geomagnetic and periodic components, in modelling the variations of daily GPS-vTEC observations of Ethiopian ionosphere in the years 2012, 2014 and 2016. Specifically, we focused on modelling the vTEC variations by incorporating the solar, geomagnetic and periodical components in the proposed model. The Bayesian statistics based approach is implemented to estimate ten model parameters over Arbaminch in the year 2014. The estimated parameters are then used to predict the vTEC over Bahir Dar in the year 2012, 2014 and 2016.

The results showed that the linear model performed very well comparing to both IRI 2016 and quadratic models with the linear correlation coefficient of about 0.8 for all the considered years. We came to the conclusion that the proposed linear model could explain 80% of the daily GPS-vTEC variations of Ethiopian ionosphere.

The wavelet transformation based analysis revealed that the observed and linear model vTEC have significant common periods of variations as well as common pattern of the variations, with the period of 27 days that lasted for 150 days (from the day of year 100 to 250). Specifically, we could observe common variations with both short and long periods of variations of the linear model based vTEC and the observed GPS-vTEC during the year 2014 in Ethiopian ionospheric sector.

We also demonstrated that the geomagnetic component of the model has influenced the short-period variations significantly whereas the solar activity component has contributed to the 27 days period of variations. This has led us to infer that the geomagnetic component is the key component for modelling the short period daily variations of GPS-vTEC and the solar component has played a great role in modelling of the 27 days period of variations using the linear time series model for the Ethiopian sector in particular and the East African sector in general.

Our analysis also showed the existence of a clear seasonal dependence of performance of the linear model in describing the expected daily vTEC variations for both stations and the year we have analysed.

ACKNOWLEDGEMENT

The author is grateful for the main financial support provided by the Air Force Office of Scientific Research, Air Force material command, USAF under the award number FA9550-16-1-0070. I also appreciate Bahir Dar University, College of Science, Research and Community Service Coordination Office for partly funding my work.

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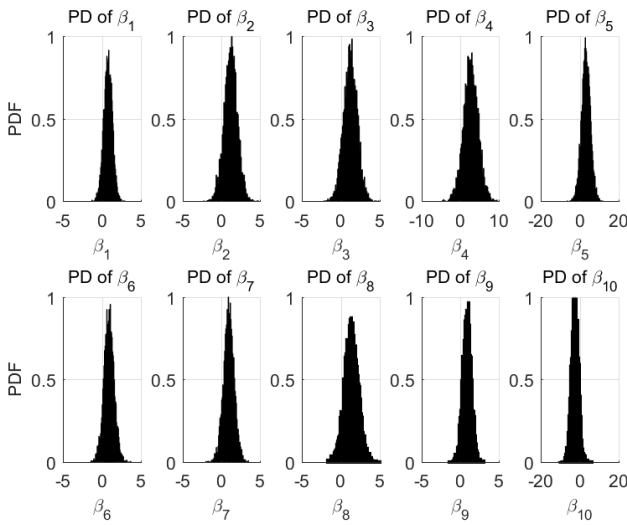


Fig. 2: Distribution of the estimated parameters over Arbaminch in the year 2014. β_1 is a constant that weights additional variations arising from the solar activity; β_2 and β_3 weight the variation due to the solar component; β_4 to β_6 due to the geomagnetic component and β_7 to β_{10} due to the periodic components.

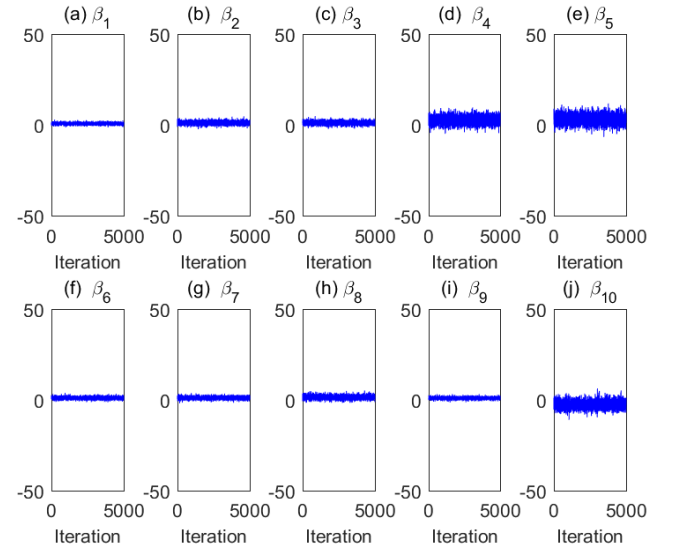


Fig. 3: Trace plot of the estimated parameters over Arbaminch for the year 2014. β_1 is a constant that weights additional variations arising from the solar activity; β_2 and β_3 weight the variation due to the solar component; β_4 to β_6 due to the geomagnetic component and β_7 to β_{10} due to the periodic components.

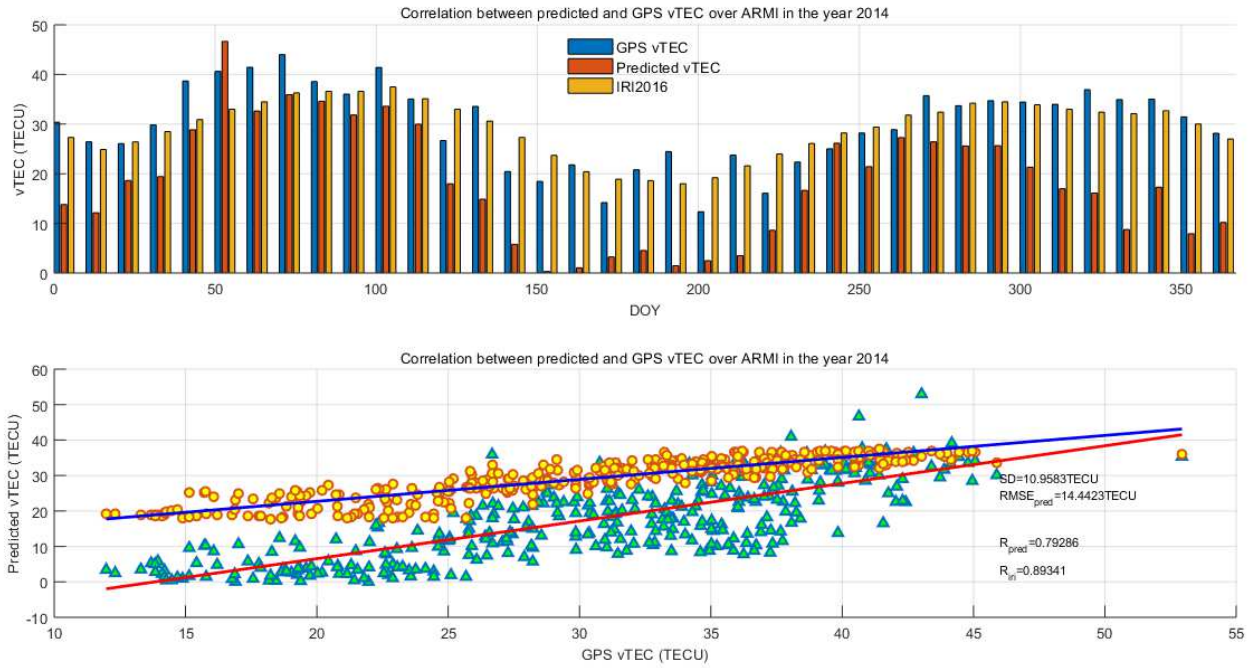


Fig. 4: Correlation between the observed and modeled vTEC over Arbaminch in the year 2014. The triangle scatter plot refers to the correlation between the observed and predicted (linear model based) vTEC whereas the circle scatter plot shows the correlation between the observed and IRI2016 model based vTEC.

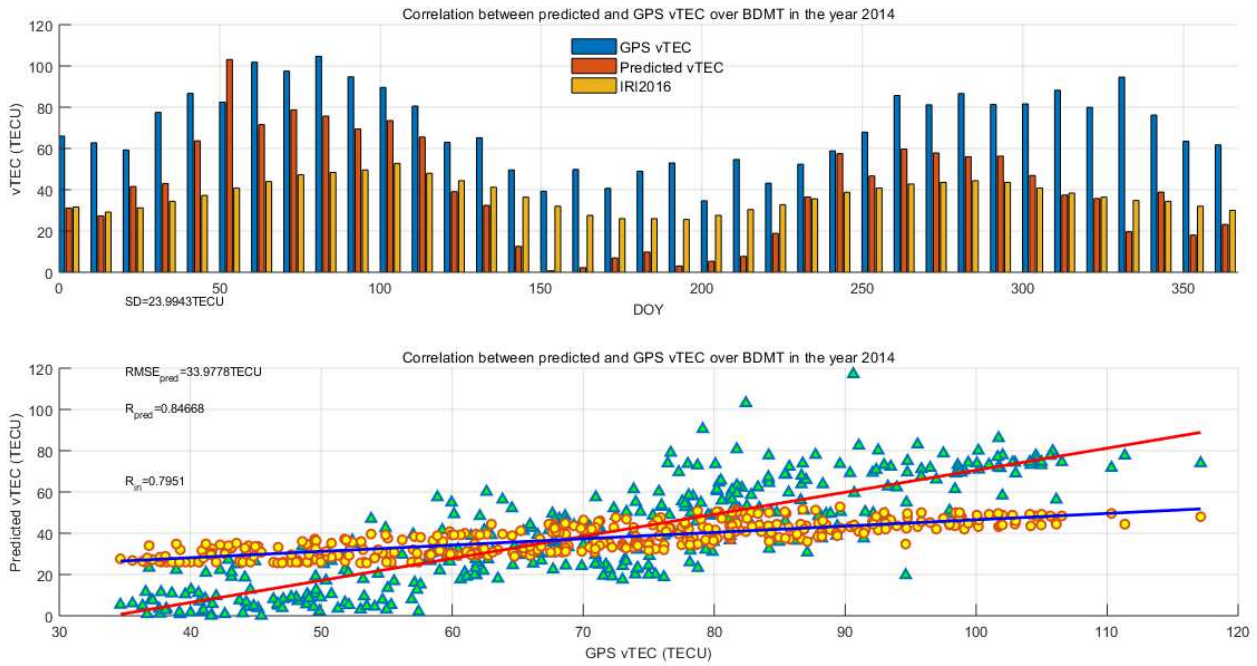


Fig. 5: Correlation between the observed and modeled vTEC and observed over Bahir Dar in the year 2014. The triangle scatter plot refers to the correlation between the observed and predicted (linear model based) vTEC whereas the circle scatter plot shows the correlation between the observed and IRI2016 model based vTEC.

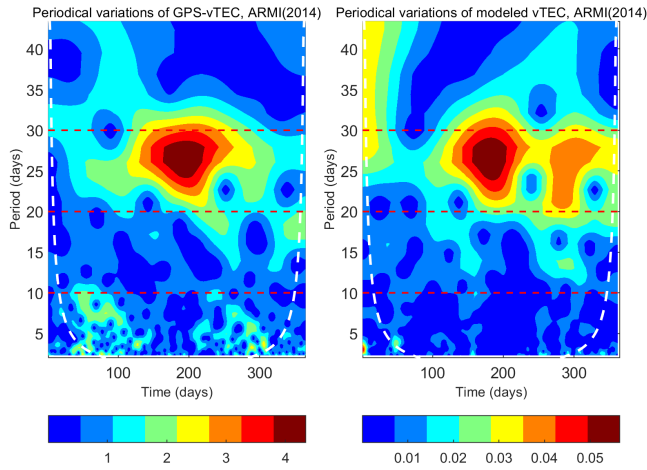


Fig. 6: Periodical variations of the GPS-vTEC (left) and the predicted vTEC over Arbaminch (ARMI) in the year 2014 (right).

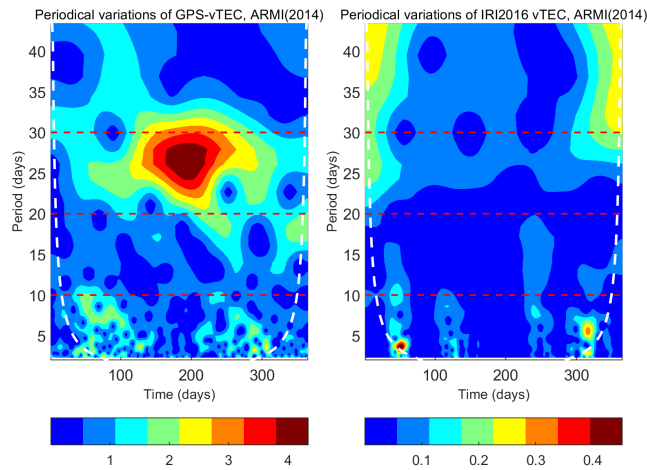


Fig. 8: Periodical variations of the GPS-vTEC (left) and the IRI2016 vTEC over Arbaminch (ARMI) in the year 2014 (right).

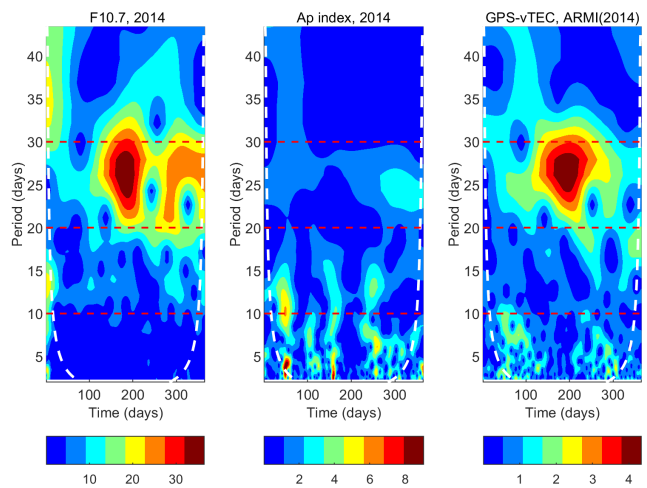


Fig. 10: Periodical variations of F10.7 (left), Ap index (middle) and GPS-vTEC over Arbaminch (ARMI) in the year 2014 (right).

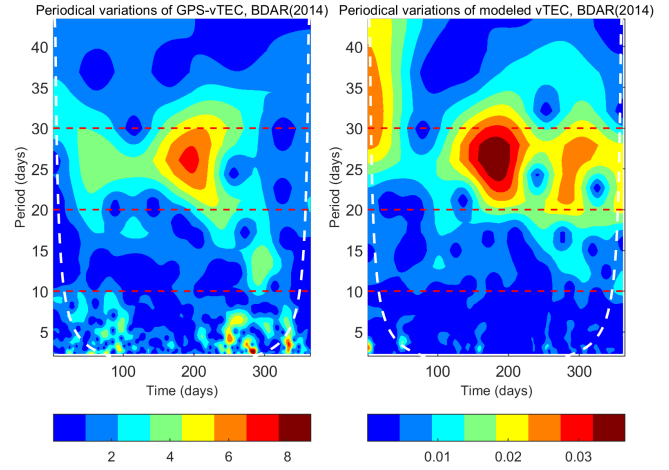


Fig. 7: Periodical variations of the GPS-vTEC (left) and the predicted vTEC over Bahir Dar (BDAR) in the year 2014 (right).

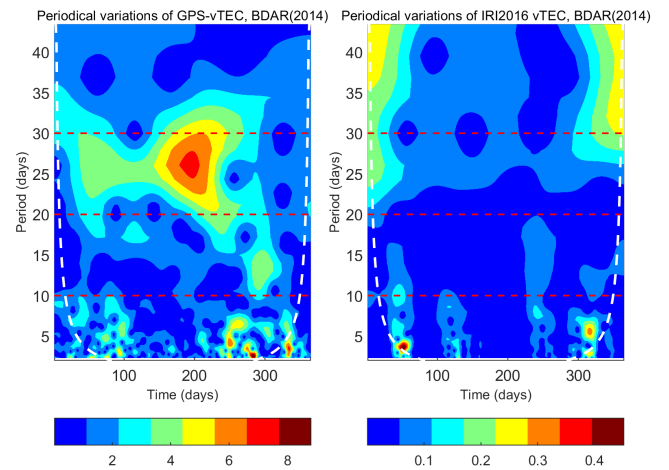


Fig. 9: Periodical variations of the GPS-vTEC (left) and the IRI2016 vTEC over Bahir Dar (BDAR) in the year 2014 (right).

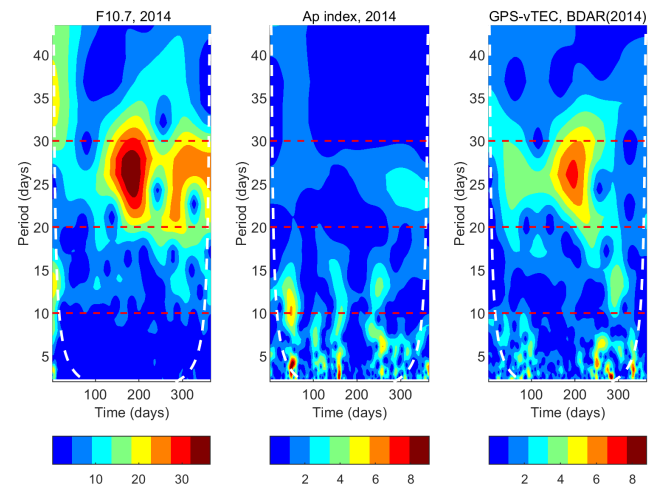


Fig. 11: Periodical variations of F10.7 (left), Ap index (middle) and GPS-vTEC over Bahir Dar (BDAR) in the year 2014 (right).

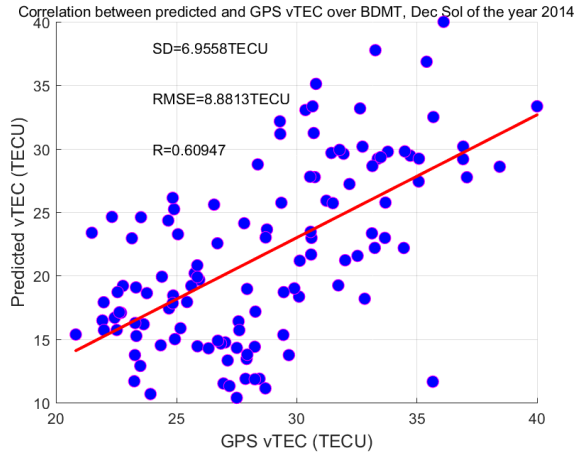


Fig. 12: Correlation between the observed and linear model based vTEC values over Bahir Dar during the December Solstice in the year 2014.

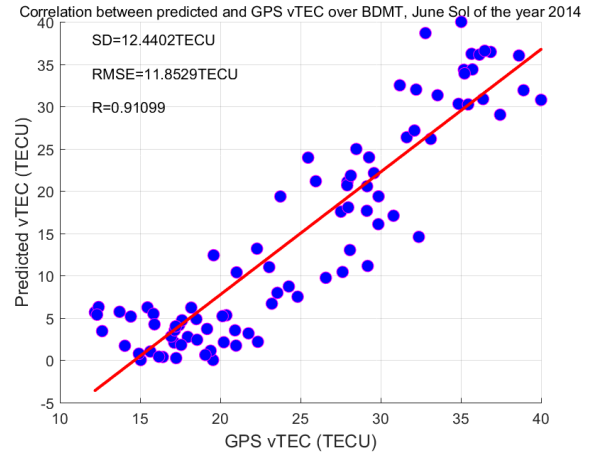


Fig. 13: Correlation between the observed and linear model based vTEC values over Bahir Dar during the June Solstice in the year 2014.

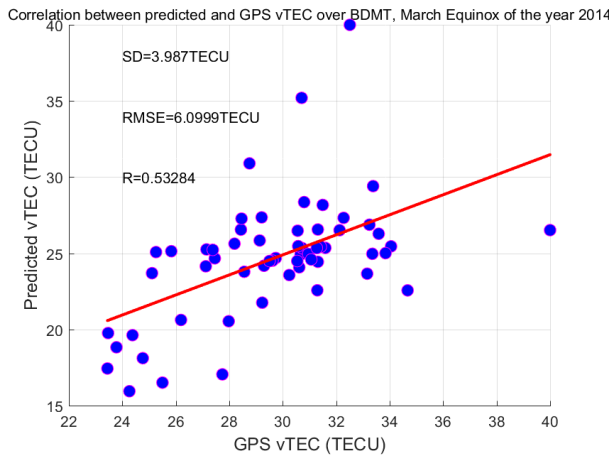


Fig. 14: Correlation between the observed and linear model based vTEC values over Bahir Dar during the March Equinox in the year 2014.

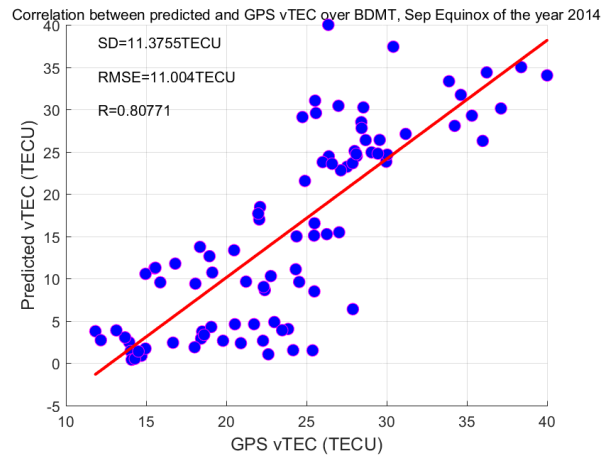


Fig. 15: Correlation between the observed and linear model based vTEC values over Bahir Dar during the September Equinox in the year 2014.

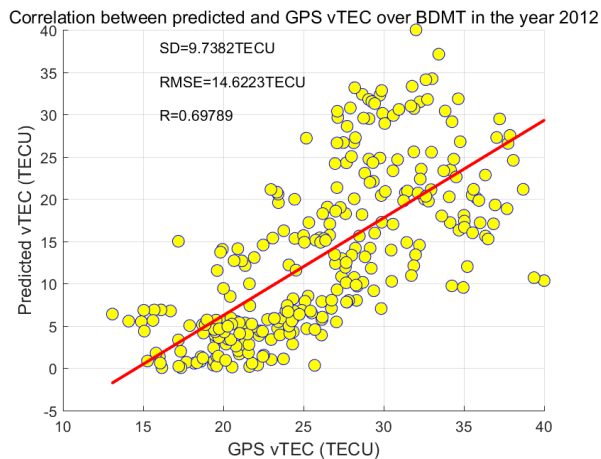


Fig. 16: Correlation between the observed and linear model based vTEC values over Bahir Dar in the year 2012.

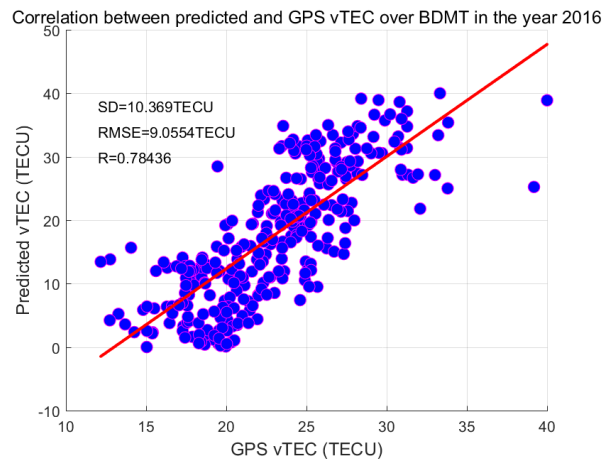


Fig. 17: Correlation between the observed and linear model based vTEC values over Bahir Dar in the year 2016.

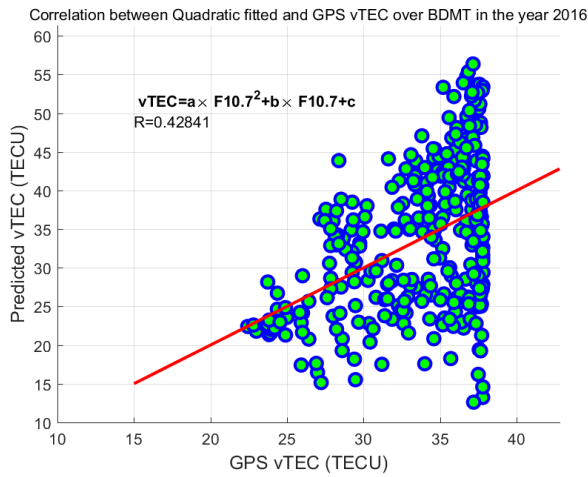


Fig. 18: Correlation between the observed and quadratic model based vTEC values over Bahir Dar in the year 2016.

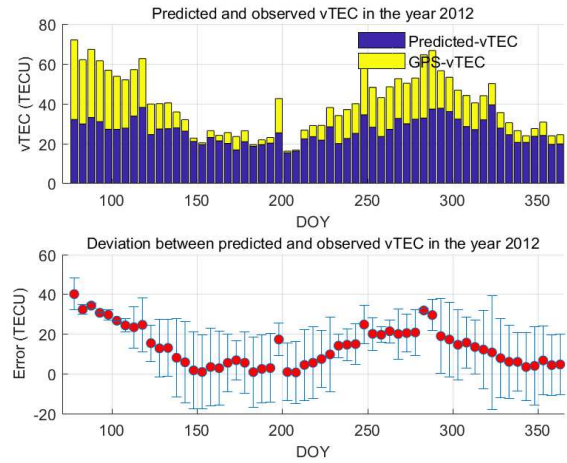


Fig. 19: Deviation between the observed and linear model based vTEC values over Bahir Dar in the year 2012. The lower panel shows the error bar illustrating the deviation of the linear model based vTEC values from the observed GPS vTEC in the year 2012 over Bahir Dar.

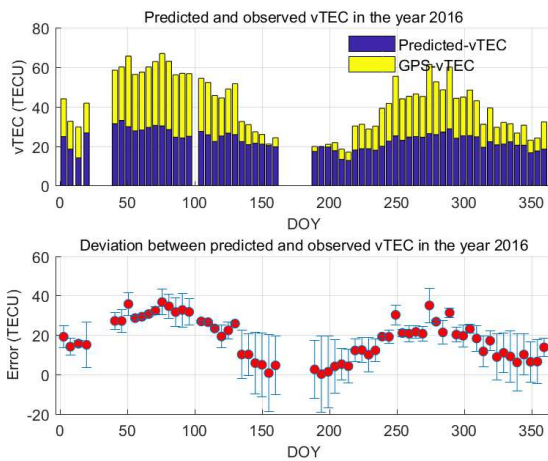


Fig. 20: Deviation between the observed and linear model based vTEC values over Bahir Dar in the year 2016. The lower panel shows the error bar illustrating the deviation of the linear model based vTEC values from the observed GPS vTEC in the year 2016 over Bahir Dar.

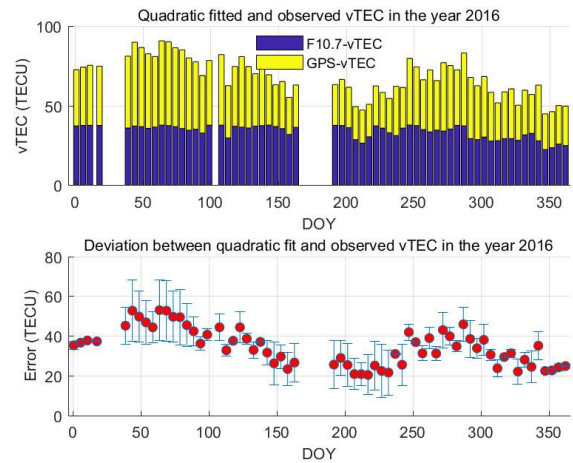


Fig. 21: Deviation between the observed and quadratic model based vTEC values over Bahir Dar in the year 2016. The lower panel shows the error bar illustrating the deviation of the quadratic model based vTEC values from the observed GPS vTEC in the year 2016 over Bahir Dar.